HYPER VERSION

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Heavy quark limit in the light front quark model

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ABSTRACT: An explicit relativistic light-front quark model is presented which gives the momentum transfer dependent form factors of weak hadronic currents among heavy pseudoscalar and vector mesons in the whole accessible kinematical region $0 \leq q^2 \leq q^2_{max}$. It is shown that in the limit of infinite masses of active quarks these form factors can be expressed in terms of universal Isgur- Wise function. The explisit expression for this function is obtained. It is shown that neglect of pair creation from the vacuum in calculations of form factors does not violate Luke's theorem.

KEYWORDS: Nonperturbative Effects, Heavy Quarks Physics, Weak Decays, Phenomenological Models.

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1. Introduction

One of the most interesting subjects in the investigation of the Standard Model is the study of CP violation effects and the determination of electroweak theory fundamental parameters. In the coming decade the main efforts in this direction will be applied in the heavy quark sector. A fundamental problem for theory is to extract data at quark level from experiments that involve hadrons.

Since in the infinite quark mass limit the spins of the heavy and light degrees of freedom decouple QCD experiences great simplifications. A new $SU(2N_h)$ spin-flavour symmetry that QCD reveals for N_h heavy-quark species, [1, 2, 3] appears. This symmetry, which is not manifest in the original QCD lagrangian, becomes explicit in an effective field theory, the so-called heavy-quark effective theory [4, 5, 6]. In this limit Isgur and Wise have derived many simple and appealing relations and normalization conditions for various hadronic matrix elements. For semileptonic transition between two heavy mesons they have got that all the assosiated hadronic form factors can be expressed in terms of single universal function $\xi(y)$ (where $y = u_1 \cdot u_2$ and $u_{1\mu},\ u_{2\mu}$ are the four-velocities of the heavy meson before, after the transition respectively), the Isgur-Wise function. In order to make direct connection between heavy hadron and the corresponding quark amplitudes we need knowledge of $\xi(y)$. It only depends on the transfer of four-velosities of the heavy mesons and is normalized at zero recoil. This single form factor incorporates all of the effects of the interaction between the heavy and light degrees of freedom. Its theoretical understanding is therefore of great interest. Since this function is sensetive to the effects of QCD at large distances, it cannot be calculated in pertubation theory.

The statement that the dynamics of processes involving transitions among (infinitely) heavy quarks depends only on the velocity transfer has some interesting consequences. Although the Isgur-Wise function itself is not determined from symmetry, restrictive relations between various hadronic form factors arise. In this work we use them to test the consistency of light front (LF) constituent quark model which is presented in a series of papers [7, 8, 9].

The main assumption of this approach was neglect of pair creation from the vacuum in calculation of the weak decay form factors. It resulted in slight dependence of the form factors upon the choice of the reference frame. Though the heavy quark limit in the relativistic light-front quark model has already been studied [10], it is a good consistent check to show that this dependence vanishes in this limit. The investigation of $\frac{1}{m_Q}$ expansion of matrix elements of hadronic currents (where m_Q is the mass of heavy quark) can shed light on how small pair creation contribution is for heavy-to-heavy transitions. The new result in this work is that above mentioned procedure does not violate Luke's theorem [11], which deals with the first order corrections to the infinite quark mass limit.

The plan of the paper is as follows. In sections 2, 3, 4 we describe our approach for the calculation of the weak meson form factors and show that in the limit of infinite masses of the active quarks our results obey the pattern of heavy quark symmetry. In section 5 we show that the expressions for the form factors obtained in [9] satisfy Luke's theorem. In section 6 the numerical results for the Isgur-Wise function are presented and compared with the results of other approaches. Section 7 contains a brief summary.

2. Kinematics

Herebelow, we denote by P_1,P_2 and M_1, M_2 the 4-momentum and masses of the parent and daughter mesons, respectively. The meson states are denoted as $|P>$ for a pseudoscalar state and $|P,\varepsilon>$ for a vector state, where ε is the polarization vector, satisfying $\varepsilon \cdot P = 0$. The 4-momentum transfer q is given by $q = P_1 - P_2$ and the momentum fraction r is defined as

$$r = \frac{P_2^+}{P_1^+} = 1 - \frac{q^+}{P_1^+} \,. \tag{2.1}$$

We work in the rest frame of the parent meson. The *3*-momentum $\vec{P}_2$ of the final meson is in the plane 1-3, so that $\mathbf{q}_\perp \neq 0$. We denote the angle between $\vec{P}_2$ and the 3-axis by α. Then, it can be easily verified that

$$q^2 = (1-r)(M_1^2 - \frac{M_2^2}{r}) - \frac{q_\perp^2}{r} \,, \tag{2.2}$$

$$q_\perp^2 = M_2^2(y^2-1)\sin^2\alpha \,, \tag{2.3}$$

where the "velocity transfer" y is defined as $y = u_1 u_2$ with u_1 and u_2 being the 4-velocities of the initial and final mesons. The relation between y and q^2 is given by

$$y = \frac{M_1^2 + M_2^2 - q^2}{2M_1 M_2}\,. \tag{2.4}$$

The momentum fraction r is invariant under the boosts along the 3-axis and the rotations around this axis but depends explicitly on the recoil direction. Solving eq. (2.2) for r one obtains

$$r(q^2,\alpha) = \zeta(y + \sqrt{y^2-1}\cos\alpha)\,, \tag{2.5}$$

where $\zeta = \frac{M_2}{M_1}$. Note that at the point of zero recoil r does not depend on α, $r(q^2_{max}) = \zeta$.

From Lorenz invariance one finds the form factor decomposition of matrix elements of the vector and axial currents. We define the form factors of the $P_1(Q_1\bar{q}) \to P_2(Q_2\bar{q})$ transitions between the ground state S-wave mesons in the usual way. The amplitude $< P_2|V_\mu|P_1 >=< P_2|\bar{Q}_2\gamma_\mu Q_1|P_1 >$ can be expressed in terms of two form factors

$$< P_2|V_\mu|P_1 >= \left(P_\mu - \frac{M_1^2 - M_2^2}{q^2}q_\mu\right)F_1(q^2) + \frac{M_1^2 - M_2^2}{q^2}q_\mu F_0(q^2)\,, \tag{2.6}$$

where $P = P_1 + P_2$. There is one form factor for the amplitude $< P_2, \varepsilon, |V_\mu|P_1 >$

$$< P_2, \varepsilon|V_\mu|P_1 >= \frac{2i}{M_1 + M_2}\varepsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}P_1^\alpha P_2^\beta V(q^2)\,, \tag{2.7}$$

and three independent form factors for the amplitude

$$\begin{aligned}
< P_2, \varepsilon|A_\mu|P_1 > &= < P_2, \varepsilon|\bar{Q}_2\gamma_\mu\gamma_5 Q_1|P_1 >\,,\\
< P_2, \varepsilon|A_\mu|P_1 > &= \Big((M_1 + M_2)\varepsilon^{*\mu}A_1(q^2) - \frac{\varepsilon^* q}{M_1 + M_2}(P_1 + P_2)_\mu A_2(q^2) -\\
&\quad -2M_2\frac{\varepsilon^* q}{q^2}q_\mu A_3(q^2)\Big) + 2M_2\frac{\varepsilon^* q}{q^2}q_\mu A_0(q^2)\,,
\end{aligned} \tag{2.8}$$

where $A_0(0) = A_3(0)$ and $A_3(q^2)$ is given by the linear combination

$$A_3(q^2) = \frac{M_1 + M_2}{2M_2}A_1(q^2) - \frac{M_1 - M_2}{2M_2}A_2(q^2)\,. \tag{2.9}$$

In the case of heavy–to–heavy transitions, in the limit in which the active quarks have infinite mass, all the form factors are given in terms of a single function $\xi(y)$, the Isgur–Wise form factor. In the realistic case of finite quark masses these relations are modified: each form factor depends separately on the dynamics of the process.

The relations between the form factors arising in this limit read

$$F_1 = V = A_0 = A_2 = \frac{M_1 + M_2}{2\sqrt{M_1 M_2}}\xi(y), \tag{2.10}$$

$$F_0 = A_1 = \frac{1 + y}{M_1 + M_2}\sqrt{M_1 M_2}\xi(y)\,. \tag{2.11}$$

3. The matrix elements of the vector and axial currents

In ref. [9] it was shown that one can determine all of the form factors by taking matrix elements of the good components of the weak currents. These components may be written in the form [1]

$$\begin{aligned}
J_V(q^2,r) &= < P_2|V^+|P_1 > \\
&= \sqrt{M_1M_2}\int_0^{r(\alpha)}\frac{dx}{2x}\int\frac{d^2k_\perp}{m^2}\phi_2(x',k_\perp'^2)\phi_1(x,k_\perp^2)\cdot I_V\,, && (3.1)\\
J_{V,+1}(q^2,q_\perp) &= < P_2,\varepsilon(+1)|V^+|P_1 > \\
&= \sqrt{M_1M_2}\int_0^{r(\alpha)}\frac{dx}{2x}\int\frac{d^2k_\perp}{m^2}\phi_2(x',k_\perp'^2)\phi_1(x,k_\perp^2)\cdot I_{V,+1}\,, && (3.2)\\
J_{A,0}(q^2,r) &= < P_2,\varepsilon(0)|A^+|P_1 > \\
&= \sqrt{M_1M_2}\int_0^{r(\alpha)}\frac{dx}{2x}\int\frac{d^2k_\perp}{m^2}\phi_2(x',k_\perp'^2)\phi_1(x,k_\perp^2)\cdot I_{A,0}\,, && (3.3)\\
J_{A,+1}(q^2,q_\perp) &= < P_2,\varepsilon(+1)|A^+|P_1 > \\
&= \sqrt{M_1M_2}\int_0^{r(\alpha)}\frac{dx}{2x}\int\frac{d^2k_\perp}{m^2}\phi_2(x',k_\perp'^2)\phi_1(x,k_\perp^2)\cdot I_{A,+1}\,, && (3.4)
\end{aligned}$$

where I_V, $I_{V,+1}$, $I_{A,\rho}$, $(\rho = 0,+1)$ are contributions of the Dirac currents and quark spin structures:

$$\begin{aligned}
I_V &= \frac{\mu_1\mu_2}{2mM_1M_2}Tr[R^+_{00}(x',k'_\perp\bar\lambda,\lambda_2)\bar u(\bar p_2,\lambda_2)\gamma^+u(p_1,\lambda_1)R_{00}(x,k_\perp,\lambda_1,\bar\lambda)]\,, && (3.5)\\
I_{V,+1} &= \frac{\mu_1\mu_2}{2mM_1M_2}Tr[R^+_{1+1}(x',k'_\perp,\bar\lambda,\lambda_2)\bar u(\bar p_2,\lambda_2)\gamma^+u(p_1,\lambda_1)R_{00}(x,k_\perp\lambda_1,\bar\lambda)]\,, && (3.6)\\
I_{A,\rho} &= \frac{\mu_1\mu_2}{2mM_1M_2}Tr[R^+_{1,\rho}(x',k'_\perp,\bar\lambda,\lambda_2)\bar u(\bar p_2,\lambda_2)\gamma^+\gamma_5u(p_1,\lambda_1)R_{00}(x,k_\perp,\lambda_1,\bar\lambda)]\,, && (3.7)
\end{aligned}$$

with

$$\mu_1 = [M_{10}^2-(m_1-m)^2]^{1/2}\,,\qquad \mu_2 = [M_{20}^2-(m_2-m)^2]^{1/2}\,, \tag{3.8}$$

and

$$\begin{aligned}
M_{10}^2 &\equiv M_{10}^2(x,k_\perp^2) = \frac{m^2+k_\perp^2}{x}+\frac{m_1^2+k_\perp^2}{1-x}\,, && (3.9)\\
M_{20}^2 &\equiv M_{20}^2(x',k_\perp'^2) = \frac{m^2+k_\perp'^2}{x'}+\frac{m_2^2+k_\perp'^2}{1-x'}\,. && (3.10)\\
& && (3.11)
\end{aligned}$$

[1]The spectator quark carries the fraction x of the plus component of the meson momentum, while the heavy quark carries the fraction $1-x$. In what follows it is assumed that the variables r, q^2, and $q_\perp^2$ are related by eq. (2)

In these equations m_1 and m_2 are the masses of active quarks, m is the mass of the quark–spectator, $x' = \frac{x}{r}$ and $\mathbf{k}'_\perp = \mathbf{k}_\perp + x'\mathbf{q}_\perp$. In our kinematics

$$k'_1 = k_1 - x'\sqrt{y^2-1}M_2 \sin\alpha\,, \qquad k'_2 = k_2. \tag{3.12}$$

The spin wave functions R_{J,J_3} can be found in ref. [12]. The wave function $\phi(x,k_\perp^2)$ can be related to wave function $\chi(x,k_\perp^2)$ from [9] by a simple formula

$$\phi(x,k_\perp^2) = 2\sqrt{M_i m m}\chi(x,k_\perp^2)\,.$$

It has been shown in [7] that phenomenological wave function $\chi(x,k_\perp^2)$ can be written in terms of equal-time wave function $w(k^2)$ normalized according to

$$\int_0^\infty dk\ k^2\ w^2(k^2) = 1, \tag{3.13}$$

where the fraction x is replaced by the relative longitudinal momentum $k_3^{(1)}$ of two quarks in the parent meson defined as

$$k_3^{(1)} = \left(x - \frac{1}{2}\right) M_{10} + \frac{m_1^2 - m^2}{2M_{10}}\,, \tag{3.14}$$

and the fraction x' is replaced by the relative longitudinal momentum $k_3^{(2)}$ of two quarks in daughter meson

$$k_3^{(2)} = \left(x' - \frac{1}{2}\right) M_{20} + \frac{m_2^2 - m^2}{2M_{20}}\,. \tag{3.15}$$

Explicitly, one has [7]

$$\phi_i(x,k_\perp^2) = \frac{\sqrt{M_i m m}}{1-x}\frac{\sqrt{M_{i0}[1-(m_i^2-m^2)^2/M_{i0}^4]}}{\sqrt{M_{i0}^2-(m_i-m)^2}}\ \frac{w_i(k^2)}{\sqrt{4\pi}}\,, \tag{3.16}$$

with $k^2 \equiv k_\perp^2 + k_3^2$. Noting that

$$\bar u(p_2,\lambda_2)\gamma^+ u(p_1,\lambda_1) = \sqrt{4p_1^+p_2^+}\delta_{\lambda_2\lambda_1} \qquad \bar u(p_2,\lambda_2)\gamma^+\gamma_5 u(p_1,\lambda_1) = \sqrt{4p_1^+p_2^+}\varphi^+_{\lambda_2}\sigma_3\varphi_{\lambda_1}\,,$$

where φ_λ are the Pauli spinors, we obtain

$$I_V = \frac{1}{x' m M_2}[\mathcal{A}_1\mathcal{A}_2 + \mathbf{k}_\perp\mathbf{k}'_\perp]\,, \tag{3.17}$$

$$I_{V,+1} = -\frac{1}{\sqrt{2}m x' M_2}[k'_1\mathcal{A}_1 - k_1\mathcal{A}_2 + \frac{2k_2^2(k'_1-k_1)}{M_{20}+m+m_2}]\,, \tag{3.18}$$

$$I_{A,+1} = \frac{1}{\sqrt{2}m x' M_2}[(2x'-1)k'_1\mathcal{A}_1 + k_1\mathcal{A}_2 + 2k'_1\frac{\mathcal{A}_1\mathcal{B} + \mathbf{k}_\perp\mathbf{k}'_\perp}{M_{20}+m+m_2}]\,, \tag{3.19}$$

$$I_{A,0} = \frac{1}{m x' M_2}[\mathcal{A}_1\mathcal{A}_2 + (1-2x')\mathbf{k}_\perp\mathbf{k}'_\perp + \frac{2\mathcal{A}_1 k'^2_\perp - 2\mathcal{B}(\mathbf{k}_\perp\mathbf{k}'_\perp)}{M_{20}+m+m_2}]\,, \tag{3.20}$$

with

$$\mathcal{A}_1 = xm_1 + (1-x)m\,, \qquad \mathcal{A}_2 = x'm_2 + (1-x')m\,, \tag{3.21}$$

and

$$\mathcal{B} = (1-x')m - x'm_2. \tag{3.22}$$

To extract a leading term in $(\frac{1}{m_Q})$ expansions of good components of the currents it should be noted that the effective wave function $\phi_i(x, k_\perp^2)$ has the maximum at $x = \frac{m}{m_i}$ and its width $\approx \frac{m}{m_i}$ [13]. Thus for heavy-to-heavy transitions quantities $\frac{m}{M_1}$, $\frac{m}{M_2}$, $\frac{M_1-m_1}{M_1}$, $\frac{M_2-m_2}{M_2}$, x, x', should be treated as small parameters. In the leading term of this expansion we get

$$\begin{aligned}
I_V^{(0)} &= \frac{1}{x'mM_2}[(m+xM_1)(m+x'M_2) + \mathbf{k}_\perp \mathbf{k}'_\perp] \\
&= \left(1 + \frac{(v, u_1+u_2)}{1+y}\right)(u_1+u_2)^+\,, && (3.23)\\
I_{V,+1}^{(0)} &= \frac{1}{\sqrt{2}mx'M_2}(k'_1(xM_1+m) - k_1(x'M_2+m)) \\
&= -i\epsilon^{+\alpha\beta\gamma}(u_{1\alpha}u_{2\beta}\varepsilon^*_\gamma(+1) + u_{1\alpha}v_\beta\varepsilon^*_\gamma(+1) + v_\alpha u_{2\beta}\varepsilon^*_\gamma(+1))\,, && (3.24)\\
I_{A,+1}^{(0)} &= -\frac{1}{\sqrt{2}mx'M_2}(k'_1(xM_1+m) - k_1(x'M_2+m)) \\
&= -u_2^+(v\varepsilon^*(+1) + u_1\varepsilon^*(+1)) - v^+(u_1\varepsilon^*(+1)) + u_1^+(v\varepsilon^*(+1))\,, && (3.25)\\
I_{A,0}^{(0)} &= \frac{1}{x'mM_2}[(m+xM_1)(m+x'M_2) + \mathbf{k}_\perp \mathbf{k}'_\perp] \\
&= \left(1 + \frac{(v, u_1+u_2)}{1+y}\right)(u_1+u_2)^+\,, && (3.26)
\end{aligned}$$

where index (0) denotes the leading term of the expansion, u_1,u_2 are four-velocities of parent and daughter mesons respectively and v is four-velocity of spectator-quark

$$v = (v^-, v^+, v_\perp) = (\frac{m^2+k_\perp^2}{xM_1m}, \frac{xM_1}{m}, \frac{k_\perp}{m})\,. \tag{3.27}$$

$\varepsilon(\lambda), (\lambda = \pm 1, 0)$ are polarization vectors:

$$\varepsilon(0) = \frac{1}{M_2}\left(\frac{-M_2^2 + P_\perp^2}{P^+}, P^+, P_\perp\right), \quad \varepsilon(\pm 1) = \left(\frac{2}{P^+}\mathbf{P}_\perp \epsilon_\perp(\pm 1), 0, \epsilon_\perp(\pm 1)\right), \tag{3.28}$$

where

$$\epsilon_\perp(\pm 1) = \mp\frac{1}{\sqrt{2}}(1, \pm i)\,.$$

Noting that

$$(k_i^2)^{(0)} = m^2[(vu_i)^2 - 1], \tag{3.29}$$

$$\left(\frac{\sqrt{M_i}}{1-x}\frac{\sqrt{M_{i0}[1-(m_i^2-m^2)^2/M_{i0}^4]}}{\sqrt{M_{i0}^2 - (m_i-m)^2}}\right)^{(0)} = \frac{\sqrt{2vu_i}}{\sqrt{1+vu_i}}\,, \tag{3.30}$$

we may write a leading term in $(\frac{1}{m_Q})$ expansion of the effective wave function $\phi_i(x, k_\perp^2)$

$$\left(\phi(x, k_\perp^2)\right)^{(0)} = \phi^{(0)}(vu_i) = m^{3/2}\frac{\sqrt{2vu_i}}{\sqrt{1+vu_i}}\frac{w^{(0)}(m[(vu_i)^2-1])}{\sqrt{4\pi}}, \qquad (3.31)$$

where

$$w^{(0)}(m^2[(vu_i)^2-1]) = \lim_{m_i\to\infty} w_i(k_i^2). \qquad (3.32)$$

In the rest frame of meson when the heavy quark mass is infinite the inner momentum $\vec{\mathbf{k}}$ is equal to *3*-momentum of light quark and normalization condition for $w^{(0)}(m^2\vec{v}^2)$ is

$$m^3\int\frac{d^3\vec{v}}{4\pi}[w^{(0)}(m^2\vec{v}^2)]^2 = 1. \qquad (3.33)$$

It can be easily verified that $\dfrac{dxd^2k_\perp}{2xm^2} = \dfrac{d^3v}{2v^0}$. Using eqs. (3.23)-(3.26), (3.31) and an obvious fact that

$$\int\frac{d^3v}{2v^0}\phi^{(0)}(vu_1)\phi^{(0)}(vu_2)v_\alpha = \int\frac{d^3v}{2v^0}\phi^{(0)}(vu_1)\phi^{(0)}(vu_2)\frac{(v, u_1+u_2)}{2(1+y)}(u_1+u_2)_\alpha, \quad (3.34)$$

we may write for the good components of the currents the following expressions:

$$J_V^{(0)}(q^2, r) = \sqrt{M_1M_2}\xi(y)(1+\frac{rM_1}{M_2}), \qquad (3.35)$$

$$J_{V,+1}^{(0)}(q^2, q_\perp) = \sqrt{\frac{M_1}{M_2}}\xi(y)q_\perp, \qquad (3.36)$$

$$J_{A,0}^{(0)}(q^2, r) = \sqrt{M_1M_2}\xi(y)(1+\frac{rM_1}{M_2}), \qquad (3.37)$$

$$J_{A,+1}^{(0)}(q^2, q_\perp) = \frac{1}{\sqrt{2}}\sqrt{\frac{M_1}{M_2}}\xi(y)q_\perp, \qquad (3.38)$$

where $\xi(y)$ is universal Isgur-Wise function

$$\xi(y) = \int\frac{d^3v}{2v^0}\phi^{(0)}(vu_1)\phi^{(0)}(vu_2)\left(1+\frac{(v, u_1+u_2)}{1+y}\right). \qquad (3.39)$$

It has the same structure as that one obtained from the analysis of the Feynman triangle diagram assuming simple exponential parametrization for the vertex functions $\phi^{(0)}(vu_i)$ [13]. This function is normalized at the point of zero recoil

$$\xi(1) = m^3\int\frac{d^3\vec{v}}{4\pi}[w^{(0)}(m^2\vec{v}^2)]^2 = 1. \qquad (3.40)$$

In case of the spinless quarks (i.e. assuming $R_{00}=1$ and $\bar{u}(\bar{p}_2)\gamma^+u(p_1) = p_1^+ + p_2^+$ in eq. (3.5)) it can be easily shown that universal form factor $\xi_{w.s.}(y)$ takes the form

$$\xi_{w.s.}(y) = \int\frac{d^3v}{2v^0}\phi^{(0)}(vu_1)\phi^{(0)}(vu_2)\sqrt{1+vu_1}\sqrt{1+vu_2}. \qquad (3.41)$$

4. The definition of the vector and axial form factors in the infinite quark mass limit

Recall [7] that the time-like LF result for the "good" component of the weak vector current $J^+ = J^0 + J^3$ coincides with the contribution of the spectator pole of the Feynman triangle diagram, which corresponds to the valence quark approximation, while the remaining part of the Feynman diagram, the so-cold Z graph, can not be expressed directly in terms of a valence quark wave function. The sum of both contributions does not depend, of course, on the choice of the frame but each contribution is frame-dependent. Therefore the time-like LF result for the form factors generally depends on the recoil direction of the daughter meson relative to the 3-axis e.g. on the choice of angles α specifying a reference frame. Fortunately Z graph does not give contribution to the leading term of $(\frac{1}{m_Q})$ expansion of plus components of weak currents and thus in this limit the form factors do not depend on this choice. To illustrate this point we reexamine an approach of ref. [7], which was used to calculate the form factors $F_1(Q^2)$ for PS–PS transitions. We first reexamine an approach of ref. [7] to calculate the form factors $F_1(q^2)$ for the $PS-PS$ transitions. Equation (2.6) for the plus component of the vector current yields only one constraint for the two formfactors $F_1(q^2)$ and $F_0(q^2)$. In order to revert eq. (2.6) the matrix element of the current was calculated in [7] in two reference frames having the 3-axis parallel and anti-parallel to the *3*-momentum of the daughter meson. This corresponds to the choice $\alpha_1 = 0, \alpha_2 = \pi$ where the angles α_i have been defined in section 2. But we can use two other frames. Specifying these frames by the two arbitrary angles α_1 and α_2 we write eq. (35) of ref. [7] as

$$F_1(q^2) = \frac{(1-r(\alpha_2))J_V(q^2, r(\alpha_1)) - (1-r(\alpha_1))J_V(q^2, r(\alpha_2))}{2M_1(r(\alpha_1) - r(\alpha_2))} \,. \tag{4.1}$$

In the leading term of $\frac{1}{m_Q}$ expansion we get for the form factor $F_1(q^2)$

$$F_1(q^2) = \frac{(1-r(\alpha_2))J_V^{(0)}(q^2, r(\alpha_1)) - (1-r(\alpha_1))J_V^{(0)}(q^2, r(\alpha_2))}{2M_1(r(\alpha_1) - r(\alpha_2))} = \frac{M_1+M_2}{2\sqrt{M_1M_2}}\xi(y) \,. \tag{4.2}$$

So in this limit the slight dependence of the form factor upon the α choice vanishes. Using the equations (13), (15), (48), (50) and (52) of ref. [9] we may apply the same procedure for form factors V, A_1, A_2, A_0.

$$V^{(0)}(q^2) = \frac{1}{\sqrt{2}}(1+\zeta)[\frac{\partial}{\partial q_\perp} J^{(0)}_{V,+1}(q^2, q_\perp)] = \frac{M_1+M_2}{2\sqrt{M_1M_2}}\xi(y) \,, \tag{4.3}$$

$$\begin{aligned} A_0^{(0)}(q^2) &= \frac{1}{2M_1 r} J^{(0)}_{A,0}(q^2, r) - \frac{(1-r)\zeta}{\sqrt{2}r} A_+^{(0)}(q^2, r) - \frac{(\zeta^2 - r^2)(1-r)}{2\sqrt{2}\zeta}\frac{\partial}{\partial r} A_+^{(0)}(q^2, r) \\ &= \frac{M_1+M_2}{2\sqrt{M_1M_2}}\xi(y) \,, \end{aligned} \tag{4.4}$$

$$A_1^{(0)}(q^2) = \frac{1}{M_1+M_2}\frac{\zeta}{r}[J_{A,0}^{(0)}(q^2,r)\frac{M_2}{\sqrt{2}\zeta^2}(r^2-\zeta^2)A_+^{(0)}(q^2,r)]$$
$$= \frac{1+y}{M_1+M_2}\sqrt{M_1M_2}\xi(y)\,, \tag{4.5}$$
$$A_2^{(0)}(q^2) = \frac{M_1+M_2}{\sqrt{2}M_1}(A_+^{(0)}(q^2,r)+r(1-r)\frac{\partial}{\partial r}A_+^{(0)}(q^2,r)) = \frac{M_1+M_2}{2\sqrt{M_1M_2}}\xi(y)\,, \tag{4.6}$$

where $A_+^{(0)}(q^2,r) = \frac{\partial}{\partial q_\perp}J_{A,+1}^{(0)}(q^2,q_\perp)$ and $\zeta = \frac{M_1}{M_2}$. Thus we have shown that all hadronic form factors for semileptonic transitions between two heavy mesons can be expressed in terms of a single universal function $\xi(y)$.

5. Luke's theorem

In what follows we show that LF approach results for the weak decay form factors [9] satisfy Luke's theorem. The only new parameter that enters the calculation is the difference between the meson mass and the mass of the heavy quark

$$\bar{\Lambda} = M_B - m_b = M_D - m_c = M_{D^*} - m_c\,,$$

where $M_{D^*} = M_D$ up to terms of order $\frac{1}{m_c}$ [14]. As noted by Luke [11] the hadronic matrix elements in equations (2.6)-(2.8) are not affected by $\frac{1}{m_Q}$ corrections at the zero-recoil normalization point $u_1 = u_2$. Inasmuch as the kinematical sructures for $J_{V,+1}$, $J_{A,+1}$ vanish at this point, Luke's theorem implies

$$J_V^{(1)}((M_1-M_2)^2,\zeta) = J_{A,0}^{(1)}((M_1-M_2)^2,\zeta) = 0\,, \tag{5.1}$$

where index (1) denotes leading corrections to the infinite quark mass limit. Expanding equations (3.1, 3.3) as a power series in $(\frac{1}{m_Q})$ at the point of zero recoil $u_1 = u_2 = u$ we get

$$J_V^{(1)}((M_1-M_2)^2,\zeta) = \sqrt{M_1M_2}\int_0^\zeta\int\frac{dxd^2k_\perp}{2xm^2}\cdot$$
$$\cdot\Big((\phi^{(0)}(vu))^2I_V^{(1)} + \phi^{(0)}(vu)\phi_1^{(1)}(x,k_\perp^2)I_V^{(0)} + \phi^{(0)}(vu)\phi_2^{(1)}(x',k_\perp^2)I_V^{(0)}\Big)\,, \tag{5.2}$$

$$J_{A,0}^{(1)}((M_1-M_2)^2,\zeta) = \sqrt{M_1M_2}\int_0^\zeta\int\frac{dxd^2k_\perp}{2xm^2}\cdot$$
$$\cdot\Big((\phi^{(0)}(vu))^2I_{A,0}^{(1)} + \phi^{(0)}(vu)\phi_1^{(1)}(x,k_\perp^2)I_{A,0}^{(0)} + \phi^{(0)}(vu)\phi_2^{(1)}(x',k_\perp^2)I_{A,0}^{(0)}\Big)\,, \tag{5.3}$$

where

$$I_V^{(1)} = I_{A,0}^{(1)} = -(\frac{1}{M_1}+\frac{1}{M_2})\frac{m+\bar{\Lambda}}{m}(xM_1+m)\,. \tag{5.4}$$

To treat the leading correction to $\phi^{(0)}(vu)$ one should consider the expansion of normalization condition for the effective wave function $\phi_i(x,k_\perp^2)$ equations (3.13), (3.16)

$$\int_0^1\int\frac{dxd^2k_\perp}{4xm^2}\phi_i^2(x,k_\perp^2)I_i=1\,, \tag{5.5}$$

where $I_i=\frac{1}{xmM_i}[A_i^2+k_\perp^2]$, $i=1,2$. Expanding this condition we get

$$\begin{aligned}1=&\int_0^1\int\frac{dxd^2k_\perp}{4xm^2}(\phi^{(0)}(vu))^2I^{(0)}+\\&+\int_0^1\int\frac{dxd^2k_\perp}{4xm^2}\left(2\phi^{(0)}(vu)\phi_i^{(1)}(x,k_\perp^2)I^{(0)}+(\phi^{(0)}(vu))^2I_i^{(1)}\right)+\cdots\,,\end{aligned} \tag{5.6}$$

where

$$\begin{aligned}I^{(0)}&=\frac{1}{xmM_1}((xM_1+m)^2+k_\perp^2)=2(1+vu)\,,\\I_i^{(1)}&=-2\frac{1}{M_i}\frac{m+\bar\Lambda}{m}(xM_i+m)\,,\end{aligned}$$

and the omitted terms in (5.6) will give next power corrections. Noting that

$$\int_0^1\int\frac{dxd^2k_\perp}{4xm^2}(\phi^{(0)}(vu))^2I^{(0)}=\xi(1)=1\,, \tag{5.7}$$

we get from equation (5.6) the equation for $\phi_i^{(1)}(x,k_\perp^2)$

$$\int_0^1\int\frac{dxd^2k_\perp}{4xm^2}\left(2\phi^{(0)}(vu)\phi_i^{(1)}(x,k_\perp^2)I^{(0)}+(\phi^{(0)}(vu))^2I_i^{(1)}\right)=0\,. \tag{5.8}$$

Substituting it into equations (5.2), (5.3) we obtain

$$J_V^{(1)}((M_1-M_2)^2,\zeta)=\sqrt{M_1M_2}\int_0^\zeta\int\frac{dxd^2k_\perp}{2xm^2}(\phi^{(0)}(vu))^2(I_V^{(1)}-\frac{1}{2}I_1^{(1)}-\frac{1}{2}I_2^{(1)})=0\,, \tag{5.9}$$

$$J_{A,0}^{(1)}((M_1-M_2)^2,\zeta)=\sqrt{M_1M_2}\int_0^\zeta\int\frac{dxd^2k_\perp}{2xm^2}(\phi^{(0)}(vu))^2(I_V^{(1)}-\frac{1}{2}I_1^{(1)}-\frac{1}{2}I_2^{(1)})=0\,, \tag{5.10}$$

which is in agreement with equation (5.1).

6. Results

The calculation of the Isgur-Wise function (eq. (3.39)) has been performed using equation (3.31) for meson wave function $\phi^{(0)}(vu_i)$. As for the radial wave function $w^{(0)}$ appearing in equation (3.31) we used two different ansätze. The authors of ref. [15] found, for three relativistic models [16, 17, 18], the wave functions are rather close to each other and that they are rather well reproduced by an exponential form in r:

$$w(r)=2a^{-3/2}exp(-r/a),\qquad w(k^2)=\sqrt{\frac{32}{\pi}}a^{3/2}\frac{1}{(1+a^2k^2)^2}\,, \tag{6.1}$$

with $a^{-1} = 0.75$ GeV. The light quark mass m is not well determined in the above relativistic spectroscopic models. But at the same time $\xi(y)$ and ρ^2 are not very sensitive to m. This is due to the fact that light quark is ultrarelativistic. In our calculations we used $m = 0.3$ GeV. The second ansatz for the radial wave function is the Gaussian of the Isgur-Scora-Grinstein-Wise (ISGW) model, the values of the parameters (the mass of the light constituent quark and the harmonic oscillator (HO) length) are taken from [19].

The y behaviour of the Isgur-Wise function is shown in fig. 1. The solid and dashed lines are the results of our LF calculation with the first and the second ansätze for the radial wave function respectively. For comparison, the result obtained from the analysis of the Feynman triangle diagram assuming simple exponential parametrization for the vertex functions $\phi^{(0)}(vu_i)$ [13] is shown by dotted line.

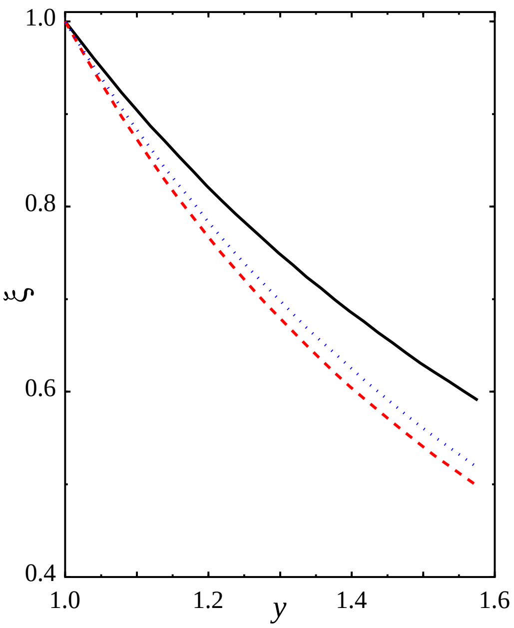

Figure 1: The Isgur-Wise function $\xi(y)$. The relation between the kinematical variables y and q^2 is given by equation (2.4). The solid and dashed lines are the results of our LF calculation with the first and the second ansätze for the radial wave function respectively. The dotted line is the result obtained from the analysis of the Feynman triangle diagram assuming simple exponential parametrization for the vertex functions $\phi^{(0)}(vu_i)$ [13].

For small, non-zero, recoil, it is conventional to write

$$\xi(y) = 1 - \rho^2(y-1) + O((y-1)^2)\,,$$

where ρ^2 is the slope of the Isgur Wise function at zero recoil. Using the first ansatz we get $\rho^2 = 1.0$, which reproduces the result of the relativistic quark models of form factors a lá Bakamjian-Thomas [15] and is in agreement with predictions of QCD fundamental methods. QCD sum rules have been used to calculate the slope parameter of the Isgur-Wise function; the results obtained by various authors are $\rho^2 = 0.84 \pm 0.02$ (Bagan *et al.* [20]), 0.7 ± 0.1 (Neubert [21]), 0.70 ± 0.25 (Blok and Shifman [22]) and 1.00 ± 0.02 (Narison [23]). UKQCD lattice calculation [24] yields 0.9 as central value, admittedly with very large error bars:

$$\rho^2 = 0.9^{+0.2+0.4}_{-0.3-0.2}\,. \tag{6.2}$$

For the second ansátz we obtained considerably higher value $\rho^2 = 1.4$, however it is in excellent agreement with calculation by Close and Wambach [25].

Two aspects [15] account of the difference between our two ansätze results. The main aspect is that the first ansätz corresponds to the relativistic spectroscopic models while the second to nonrelativistic one. In ref. [15] it was found that wave functions at rest can be strongly different in a relativistic spectroscopic model from those of nonrelativistic, although the spectrum is similar, whence lowering of ρ^2.

Another, more technical, but important aspect is that the approximation of using a variational Gaussian to approximate the wave function fails even at low recoil, e.g. in the calculation of ρ^2, especially in the context of relativistic spectroscopic equations. Calculating exactly the wave function results in appreciably lower estimate of ρ^2.

Performing the calculations with and without the effects of the Melosh composition (eqs. (3.39), (3.41)), it turns out that the effects of the Melosh rotations increase ρ^2 by 30% and 20% for the first and the second ansätze respectively; last result agrees with the conclusion of ref. [25] obtained in the zero-binding approximation.

7. Conclusions

In relativistic quark model formulated on the light front we have examined $\frac{1}{m_Q}$ expansion of form factors for semileptonic transitions between mesons. We verified that in leading term of this expansion all associated hadronic form factors can be expressed in terms of universal Isgur-Wise function. Explisit expression has been given for it. Using the fact that form factors are related to overlap integrals of hadronic wave functions we have shown that this function is normalized at the point of zero recoil. We have shown that the relativistic light front quark model of form factors, combained with a relativistic spectroscopic model to calculate the needed wave functions, reproduces the result for the slope of the universal form factor ($\rho^2 \approx 1$) of relativistic quark models a lá Bakamjian-Thomas. This result is in agreement with QCD fundamental methods. In next-to-leading term we obtained that our results obeyed Luke's theorem. Before closing, it should be reminded that in our analysis the contribution of the pair creation from the vacuum has been neglected. Therefore we can conclude that it does not contribute to the leading term of $\frac{1}{m_Q}$ expansion of the form factors and at least at the point of zero recoil to next-to-leading one.

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